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Towards objectification of measurement in an orthodox but incomplete quantum mechanics

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Abstract. The problem of objectification of measurement is investigated in orthodox quantum mechanics, that is assuming the validity of the Schrödinger equation (and the other postulates) for the composite object-plus-measuring-instrument system. It is argued that by giving physical meaning to basic entities, the positivist and the realist positions must be explicitly distinguished. It is shown that the objectification problem is satisfactorily solved within quantum mechanics for the positivist, but not for the realist. For the latter, a partially satisfactory solution is obtained by assuming that the quantum mechanical description of individual systems is incomplete and, by assuming further, that the pointer observable has definite values prior to any measurement in any quantum state (a so-called beable).

1. Introduction

We call orthodox the approach to the theory of measurement in quantum mechanics (QM) that keeps the Schrödinger equation (or equivalently, a unitary evolution operator) as the dynamical law encompassing the interaction between quantum object (subsystem 1) and the measuring instrument (MI, subsystem 2) along with the rest of the known postulates of QM (as a counterexample, see, for example, Ghirardi *et al* 1986).

In recent expositions of the orthodox approach (Busch 1991, Beltrametti *et al* 1990), measurement is viewed as consisting of three successive phases. First comes the *preparation* of the object in the (arbitrarily chosen) quantum state $|\psi_0\rangle_1$ and the MI in $|\chi_0\rangle_2$; then follows the dynamical evolution of the composite system called the *premeasurement*, that leads to a final state $|\phi\rangle_{12}$; and, finally, there is the *objectification of measurement*.

To outline the bare essentials of premeasurement, let

$$A_1 = \sum_k a_k P_1^{(k)} \qquad \left(k \neq k' \Rightarrow a_k \neq a_{k'}; \sum_k P_1^{(k)} = 1\right) \tag{1}$$

be the measured observable of the quantum object written as a Hermitian operator in spectral form. (Exact measurement is possible only if the measured observable has a purely discrete spectrum; see von Neumann (1955 p 222).) Further let

$$B_2 = \sum_k b_k Q_2^{(k)} + b_0 Q_2^{(0)} + B_2' \qquad \left(k \neq k' \Rightarrow b_k \neq b_{k'} \neq b_0; \left(\sum_k Q_2^{(k)} + Q_2^{(0)}\right) B_2' = 0\right) \tag{2}$$

be the so-called *pointer observable* of the MI (in partially spectral form). We use the terms 'pointer observable' and 'pointer positions' (b_k in a symbolical meaning) to cover a wide range of cases. The co-indexing in (1) and (2) is necessary for the interpretation of b_k as meaning that the result a_k of A_1 has been obtained. As to the initial state $|\chi_0\rangle_2$, one has

$$B_2|\chi_0\rangle_2 = b_0|\chi_0\rangle_2. \tag{3}$$

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Further let

$$|\psi_0\rangle_1 = \Sigma_k w_k^{1/2}(w_k^{-1/2}P_1^{(k)}|\psi_0\rangle_1) \tag{4}$$

be the decomposition of the initial state vector of the quantum object according to (1), into orthonormal vectors, where

$$\forall k: w_k \equiv \langle\psi_0|P_1^{(k)}|\psi_0\rangle_1 = \|P_1^{(k)}|\psi_0\rangle_1\|^2 \tag{5}$$

is the probability of a_k in $|\psi_0\rangle_1$.

To define premeasurement, we require that if the initial state $|\psi_0^{(k)}\rangle_1$ has a sharp value a_k, that is if

$$A_1|\psi_0^{(k)}\rangle_1 = a_k|\psi_0^{(k)}\rangle_1 \tag{6}$$

(equivalently, if $\langle\psi_0^{(k)}|P_1^{(k)}|\psi_0^{(k)}\rangle_1 = 1$), then the final state $|\phi^{(k)}\rangle_{12}$ of the composite system at the end of premeasurement evolution has a sharp pointer position:

$$(1\otimes B_2)|\phi^{(k)}\rangle_{12} = b_k|\phi^{(k)}\rangle_{12}. \tag{7}$$

It is of immense practical importance to achieve some permanence (in time) for relation (7): one wants to be able to avail oneself of the result for some time to follow. This is usually achieved by some process of macroscopic amplification involving irreversibility.

Denoting the composite-system unitary evolution operator by U_{12}, one has so far

$$U_{12}|\psi_0^{(k)}\rangle_1|\chi_0\rangle_2 = |\phi^{(k)}\rangle_{12}. \tag{8}$$

Since every normalized k-component $(w_k^{-1/2}P_1^{(k)}|\psi_0\rangle_1)$ of the general state $|\psi_0\rangle_1$ (see (4)) satisfies (6), and since the evolution operator is linear, (4) and (8) imply

$$U_{12}|\psi_0\rangle_1|\chi_0\rangle_2 = \Sigma_k w_k^{1/2} U_{12}(w_k^{-1/2}P_1^{(k)}|\psi_0\rangle_1|\chi_0\rangle_2)$$
$$= \Sigma_k w_k^{1/2}|\phi^{(k)}\rangle_{12} \equiv |\phi\rangle_{12}. \tag{9}$$

In view of the fact that the same expansion coefficients $w_k^{1/2}$ appear both in (4) and (9), it is easy to see that the probability distribution implied by $|\psi_0\rangle_1$ with respect to the characteristic values of A_1, is the *same* as that entailed by the final state $|\phi\rangle_{12}$ regarding the pointer positions b_k. Hence, (7) (with (8) and (6)) defines those U_{12} that give measurement (providing the pointer observable allows objectification). For an equivalent, but more detailed, premeasurement theory see Beltrametti *et al* (1990) or Mittelstaedt (1991).

Objectification depends on satisfying two requirements (cf Busch 1991):

(i) the relevant statistical state of object + MI at the end of premeasurement must be a *mixture* of substates that correspond to definite pointer positions;

(ii) there must be given a statistical mechanism leading from the final state to a definite-pointer-position substate in the mixture for every individual system in the final premeasurement state.

The orthodox theory at issue is known to fail in objectification (Mittelstaedt 1991, D'Espagnat 1976, Fine 1970) unless the idea of completeness of QM is given up (as done in section 7). One should bear in mind that, while it is indisputable that QM does give a complete description of quantum mechanical ensembles, the notion that it also describes in a complete way the *individual* systems is no more than a matter of faith (as in the well known Copenhagen position).

Any attempt at solving the objectification problem requires one to give physical meaning to some basic entities of the quantum mechanical formalism. Thorough study reveals that this actually cannot be done unless one takes a stand in the well known positivist-realist dispute. Thus, objectification of quantum measurement, that has much bearing on foundation (or interpretation) of QM, cannot, in fact, be completely decoupled from some basic philosophy. The previously mentioned no-go results are actually obtained from the position of a realist.

Of the numerous ways in which quantum measurement is treated differently than in the orthodox approach at issue (cf Sudbery 1986), the one that is perhaps the most relevant for this study is the Copenhagen instrumentalist approach (Stapp 1972). It treats measurement in terms of the laboratory instruments (described by classical physics), but it does not allow one to apply QM to the instruments. It is not so well known that there exists a broader-minded positivistic position that does allow an orthodox quantum mechanical theory of measurement and that does achieve objectification without utilizing the assumption of incompleteness of QM. It will be presented in sections 2–6 and critically discussed in the last section. Nevertheless, in this investigation, the positivist solution is only a springboard for further study.

The basic aim is to attempt to make a resolute step towards a genuine realist's objectification of measurement (at the expense of the assumption of incompleteness of QM). This is done in sections 7 and 8.

For a methodological reason, we keep philosophy out of our quantum mechanical arguments as far as possible. Hence, the positivist-realist distinction is resumed no sooner than in the last section.

A shortened version of this work (without proofs) is included in Herbut (1991).

2. Bohr's idea of macroscopic complementarity

'In his most careful writing', says Shimony in his lucid discussion (1963 p 769), 'Bohr states subtle qualifications concerning states of macroscopic objects.' Then Shimony gives an example citing Bohr (1961 p 50):

> 'The main point here is the distinction between the *objects* under investigation and the *measuring instruments* which serve to define, in classical terms, the conditions under which the phenomena appear. Incidentally, we remark that, for the illustration of the preceding considerations, it is not relevant that experiments involving an accurate control of the momentum or energy transfer from atomic particles to heavy bodies like diaphragms and shutters would be very difficult to perform, if practicable at all. It is only decisive that, in contrast to the proper measuring instruments, these bodies together with the particles would constitute the system to which the quantum mechanical formalism has to be applied.'

Shimony (1963 p 770) proceeds by giving his understanding of Bohr's idea:

> Bohr is saying that from one point of view the apparatus is described classically and from another, *mutually exclusive* point of view (my italics), it is described quantum mechanically. In other words, he is applying the principle of complementarity, which was originally formulated for microscopical phenomena, to a macroscopic piece of apparatus.

We shall call this idea of Bohr his *macroscopic complementarity*. We propose to view it as a basic principle of QM. Bohr's previously quoted text may be considered to give a rough definition of it. It is sufficient for the realization of the beginning of our programme.

Summing up Bohr's idea, we can say that the principle of macroscopic complementarity offers two mutually exclusive points of view or versions.

(i) In the *object version* the MI is viewed as a quantum object, just like any microscopic system. Then there is no restriction on the measurability of composite-system observables.

(ii) In the *instrument version*, we by definition restrict the composite-system observables to such as are relevant for having the measurement results at one's disposal. We elaborate this in section 4.

3. The concept of the cut and explanation of the role of classical apparatuses in conventional quantum mechanics

To begin with, we give a precise definition of the cut in conventional QM. Every quantum mechanical description by a quantum state has a (most often tacit) prerequisite: a division of the world into two parts, one that is the object of quantum mechanical description (briefly object or O), and one that is omitted. In the latter there is somehow stored the information on how to prepare laboratory ensembles of quantum objects in the quantum state at issue. Therefore, the second part is called the subject (S). (For more on this, see Stapp's 1972 presentation on instrumentalism.) The imagined line of division between object and subject is called the *cut*. For instance, the cut (symbolically '/') that corresponds to the final state $|\phi\rangle_{12}$ of premeasurement evolution (cf (9)) can be written as follows

$$\mathrm{O/S} = (1+2)/3. \tag{10}$$

Here the preparators (of both the quantum object and the MI) and the rest of the world are denoted as subsystem 3. They could equally well be subsystem 0, and one could write: $\mathrm{S/O} = 0/(1+2)$.

Now we discuss the role of classical apparatuses in conventional QM by answering, one by one, questions on the reason for the usual (textbook) statements.

3.1. How do the preparator and the MI 'determine' the quantum experiment?

In this study we confine ourselves to preparators that are MI. (We shall extend the orthodox theory to so-called filters as preparators elsewhere.) We measure selectively a non-degenerate value a_k of a quantum-object observable A_1 in a first-kind (that is predictive) way. As a result, the quantum object is brought into the unique (up to a phase factor) state vector $|\psi_0\rangle_1 = |a_k\rangle_1$ (cf section 1).

Preparation is the first half of a quantum mechanical experiment. Measurement is its second half. The same procedure is essentially repeated in it (we even use the same notation though now A_1 and B_2 are quite independent new observables):

The initial state $|\psi_0\rangle_1|\chi_0\rangle_2$ evolves into $|\phi\rangle_{12}$ defined by (9). Then, the new pointer observable B_2 of the MI *determines* the new collapsed state $\rho_1^{(k)}$ of the quantum object via its position b_k (assuming the objectification is valid, see (21)).

One should note that the determining role of both the preparator and the subsequent MI comes from their role as instruments (in contrast to their possible role as quantum objects according to the principle of macroscopic complementarity).

3.2. What does it mean that object and subject are 'inextricably tied up' in quantum observation?

At the end of the premeasurement evolution we have

$$|\phi\rangle_{12}=\Sigma_k w_k^{1/2}|\phi^{(k)}\rangle_{12}$$

(cf (9)), which in the object version of the MI is relevant in all its details. This 'ties up' the two subsystems inextricably through quantum correlations. Since object and MI no longer interact, we have actually distant correlations between the two subsystems (cf, for example, Vujičić and Herbut 1988).

3.3. What does it mean that there is no sharp line of division beween object and subject in quantum observation?

As previously explained, macroscopic complementarity enables us to define the cut, for example $O/S=1/(2+3)$, corresponding to the instrument version (in which the MI is part of the subject). This is a perfectly sharp definition of the cut, but it is not unique. In the object version of the MI we have the cut $O/S=(1+2)/3$. Thus, on account of macroscopic complementarity, the cut is *displaceable*, and hence the 'line of division between object and subject' that it stands for is not sharply defined.

It has, no doubt, become clear in these explanations of the role of macroscopic apparatuses in conventional QM that the proposed enrichment of standard QM by the macroscopic complementarity principle also implies an enrichment of the concept of the cut. The MI with its pointer observable and, as a rule, with a definite pointer position, is an important part of the subject, and hence it can no longer be completely omitted (as is usually done with the entire subject in conventional QM). The rest of the world, that plays a passive role, can be omitted just as in conventional QM. In other words, subsystem 3 in (10) can be restricted to the MI.

To offer an explanation why the MI is described by classical physics in conventional QM, we need a more elaborate theory of statistical states in the instrument version. But, as a prerequisite, we must propose a precise form of macroscopic complementarity.

4. Precise form of the macroscopic complementarity principle

How we have the results in $|\phi\rangle_{12}$, the final state in premeasurement evolution, at our disposal is an important question. From the operational point of view, there is no other way of control than by 'looking at the pointer position', that is by measuring B_2 (cf (2)) simultaneously with an observable C_1 on the quantum object.

To express this in precise terms, we introduce *coincidence observables* $C_1\wedge B_2$ as composite-system observables, the characteristic projectors of which are, by definition, $S_1^{(n)}\otimes Q_2^{(k)}$, where $S_1^{(n)}$ are the characteristic projectors of

$$C_1=\Sigma_n c_n S_1^{(n)} \tag{11}$$

and the 'characteristic values' that correspond to these composite-system projectors are the *pairs* (c_n, b_k), where c_n are the characteristic values of C_1 corresponding to $S_1^{(n)}$ (in the spectral form (11)).

The *subsequent measurement* on $|\phi\rangle_{12}$ that is *relevant* for the control of the results of the first measurement is the measurement of $C_1\wedge B_2$, where C_1 is an *arbitrary* observable of the quantum object, and B_2 is the pointer observable. One can measure a more general coincidence observable $C_1 \wedge f(B_2)$, where $f(B_2)$ is a Hermitian operator that is a function of B_2, for the same purpose. Then the observables $C_1 \wedge 1$ (meaning that one does not 'look' at the pointer position) are also included. The purpose in question would even allow utilizing observables of the form $g(C_1, B_2)$, where g is any functional dependence giving a Hermitian operator.

We saw (at the end of section 2) that all that remained to be specified in the concept of macroscopic complementarity is the precise set of composite-system observables to which one, by definition, restricts oneself when the MI is viewed in the instrument version. We denote this set by O_{12}, and we define it as follows.

$$O_{12} \equiv \{C_1 \wedge f(B_2) : C_1 \text{ and } f \text{ any}\}. \tag{12}$$

An observable $C_1 \wedge f(B_2)$ with f non-singular is, from the point of view of measurement, the same as $C_1 \wedge B_2$. If f is singular, then the physical meaning of $f(B_2)$ is an imprecise measurement of B_2.

5. How does the instrument version of macroscopic complementarity imply classical description of apparatuses?

In the instrument version of the MI there can exist two distinct statistical operators ρ_{12} and ρ'_{12} for the composite quantum-object-plus-MI system that cannot be distinguished by observables of the form $C_1 \wedge f(B_2)$, that is for which

$$\mathrm{Tr}_{12}\, \rho_{12}(C_1 \otimes Q_2^{(k)}) = \mathrm{Tr}_{12}\, \rho'_{12}(C_1 \otimes Q_2^{(k)}) \tag{13}$$

for any C_1 and every value of k. Hence, there is redundancy in the set S_{12} of all statistical operators of the composite system. One wonders what is the meaning of this.

5.1. Jauch's classes of statistical operators

Jauch (1964, 1968) showed that any restriction of the set of all Hermitian operators to some given subset O actually *changes our statistical state concept* as follows.

Let S be the set of all statistical operators for the system. One says that $\rho, \rho' \in S$ are *equivalent* with respect to 0, that is $\rho \sim \rho'$, if $\forall C \in O : \mathrm{Tr}\, \rho C = \mathrm{Tr}\, \rho' C$.

Then the quotient set $S/\sim$, that is the set of all equivalence classes of statistical operators, has the structure of a σ-convex set, that is that of a set of statistical states, in a natural way:

If

$$C_1, C_2, \ldots, \in S/\sim \qquad \text{and} \qquad w_1 > 0,\, w_2 > 0, \ldots, \Sigma_n w_n = 1$$

then by $\Sigma_n w_n C_n \in S/\sim$ (the so-called convex combination or σ-convex combination of elements, the latter if there is an infinite number of terms) one means the class obtainable through *arbitrary representatives* as follows. One takes $\rho^{(1)} \in C_1$, $\rho^{(2)} \in C_2, \ldots$, one defines $\Sigma_n w_n \rho^{(n)}$, and then $\Sigma_n w_n C_n$ is by definition the class to which $\Sigma_n w_n \rho^{(n)}$ belongs.

The set of observables O_{12} defined by (12) does not consist of Hermitian operators, and hence it cannot be made use of directly for the evaluation of the corresponding Jauch classes. It is easy to see that the corresponding set of Hermitian operators is

$$O_{12}^{\mathrm{H}} \equiv \{C_1 \otimes Q_2^{(k)} : C_1 \text{ any}, \forall k\} \tag{14}$$

where $Q_2^{(k)}$ are the characteristic projectors of the pointer observable B_2.

In view of (14), two composite-system statistical operators ρ_{12} and ρ'_{12} are equivalent or $\rho_{12} \sim \rho'_{12}$, that is they belong to the same Jauch class, if and only if they satisfy (13).

There is no redundancy in the set of Jauch classes. However for any evaluation one must take an arbitrary element of the class in question, and therefore working with the classes is actually not more practical than working with the statistical operators themselves. One wonders if one can do better than that.

5.2. Hybrid form as canonical form of the statistical states

With the purpose of replacing the Jauch classes of statistical operators of the composite system with single entities leaving the σ-convex structure of the quotient set intact, Herbut (1986b) investigated what properties of ρ_{12} and ρ'_{12} would make them equivalent (in the sense of (13)).

It was proved (see theorem 1 and corollary 5 of Herbut 1986b) that ρ_{12} and ρ'_{12} belong to the same Jauch class if and only if both of the following two sets of conditions are satisfied:

$$\forall k: \qquad w_k \equiv \mathrm{Tr}_{12}\, \rho_{12}(1 \otimes Q_2^{(k)}) = w'_k \equiv \mathrm{Tr}_{12}\, \rho'_{12}(1 \otimes Q_2^{(k)}) \tag{15}$$

$$\forall k,\ w_k > 0,\ w'_k > 0: \qquad \rho_1^{(k)} \equiv w_k^{-1}\, \mathrm{Tr}_2\, \rho_{12}(1 \otimes Q_2^{(k)}) = \rho_1'^{(k)} \equiv w_k'^{-1}\, \mathrm{Tr}_2\, \rho'_{12}(1 \otimes Q_2^{(k)}). \tag{16}$$

One should remember that k enumerates the pointer positions b_k (or equivalently, the quantum events $Q_2^{(k)}$ that mean taking up b_k). Thus, these necessary and sufficient conditions read: $\rho_{12} \sim \rho'_{12}$ if and only if all pointer positions are equally probable in ρ_{12} and in ρ'_{12}, and for each k value for which $w_k = w'_k > 0$, the corresponding conditional states of the quantum object are equal. (As to the concept of the conditional state $\rho_1^{(k)}$, it was investigated in full generality in Herbut 1986b, cf (5) there.)

In view of this condition, the logical question arose as to whether it was possible to redefine the Jauch classes in terms of the relevant entities:

$$M \equiv \{(\rho_1^{(k)}, w_k) : \forall k\} \tag{17}$$

putting formally $\rho_1^{(k)} \equiv 0$ whenever $w_k = 0$.

It turned out that it was possible (see theorem 4 in Herbut 1986b). The entities M were called *hybrid states* because they are half quantum mechanical and half classical discrete. They form in a natural way a σ-convex set (1986b, section 5) isomorphic to that of the Jauch classes of statistical operators. This proceeds as follows. Let

$$M_q \equiv \{(\rho_1^{(k,q)}, w_k^{(q)}) : \forall k\} \qquad v_q > 0 \qquad q = 1, 2, \ldots \qquad \Sigma_q v_q = 1.$$

Further let M defined by (17) be such that $M = \Sigma_q v_q M_q$. Then

$$\forall k: \qquad w_k = \sum_q v_q w_k^{(q)} \tag{18a}$$

$$\forall k,\ w_k > 0: \qquad \rho_1^{(k)} = \sum_q v_q (w_k^{(q)}/w_k) \rho_1^{(k,q)}. \tag{18b}$$

We have introduced the hybrid state M given by (17) in terms of entities evaluated from an *a priori* given statistical operator ρ_{12} by means of formulae (15) and (16). The set $\mathcal{M}_Q$ of all hybrid states M thus obtained (from all possible ρ_{12}) contains all possible discrete probability distributions w_k (see (17)) and, independently, all possible statistical operators (written as $\rho_1^{(k)}$) in the state space of the quantum object (observing $\rho_1^{(k)} \equiv 0$ when $w_k = 0$). This was previously proved (see theorem 4 in Herbut 1986b).

Finally, we come to the question as to whether working with the hybrid state M is more practical than with the Jauch classes. The hybrid states are of interest to us in the instrument version of macroscopic complementarity. Hence, our most general composite-system observables are of the form $C_1 \otimes Q_2^{(k)}$ (cf (14)). The expectation value is

$$\langle C_1 \otimes Q_2^{(k)} \rangle_M = \mathrm{Tr}_{12}\, \rho_{12}(C_1 \otimes Q_2^{(k)}) = w_k\, \mathrm{Tr}_1\, C_1 \rho_1^{(k)} \tag{19}$$

where ρ_{12} is an arbitrary element from the Jauch class that corresponds to the hybrid state M given by (17). It is clear from (19) that the required expectation value is easily evaluated from M, and that it is performed in terms of the canonical entities w_k and $\rho_1^{(k)}$ that are common to all elements of the Jauch class in question. Hence, one cannot, in general, evaluate $\langle C_1 \otimes Q_2^{(k)} \rangle_M$ in a more simple way than by means of (19).

5.3. Why can the apparatuses be described by classical physics?

It will require a more elaborate argument to show that the classical discrete probabilities $\{w_k : \forall k\}$ contained in the hybrid states M (cf (17)) lead, in a more or less straightforward way, to a classical statistical description of the MI. Here we want only to stress that the former represent the basic step from a quantum mechanical towards a classical description and that they emerge naturally, as a consequence of having given a precise definition of the instrument version of macroscopic complementarity in terms of the set of coincidence observables O_{12} (cf (12)).

Since the pointer observable B_2 plays a key role in O_{12}, it is, in the long run, one of the basic concepts responsible for the appearance of classical description. The important question arises as to how one should select the pointer observable when a quantum system that we call an MI is given. There are three mutually unconnected partial answers.

(i) The pointer observable should be a macroscopic variable in the sense of von Neumann (1955, section V.4), that is one from a special set of mutually compatible observables.

(ii) The observable B_2 should be an effective superselection observable, that is due to the vast number of atoms in a macroscopic environment of the pointer (or rather of the relevant part of the MI carrying the pointer). This point of view was investigated, for example, in the articles of Zurek (1982), Joos and Zeh (1985).

(iii) The pointer observable should be a beable (Bell 1987). This is actually a hidden-variable approach. It will be elaborated to some extent later.

A further contribution to a solution of the important problem of how classical physics follows from QM is to be obtained when more light is shed on the mutual relationships between these three views about a classical variable like the pointer observable.

Finally, it should be pointed out that the natural appearance of the classical description on the subject side of the cut (cf section 3) may also be significant for the quantum mechanical theory of some molecular systems. Namely, in some cases symmetry breaking, for example breaking of parity in molecules with definite chirality, can be explained in no other way than by the influence of classical surroundings (cf Primas 1983).

6. Objectification in terms of the hybrid states

One may wonder if the hybrid states are decomposable into definite-pointer-position states. Prior to an answer, what the latter means in the hybrid-state formalism must be clarified.

Lemma. A hybrid state is a statistical state with a definite pointer position b_{k_0} if and only if it has the form

$$M^{(k_0)} = \{(\delta_{k,k_0}\rho_1, \delta_{k,k_0}) : \forall k\} \tag{20}$$

where ρ_1 is a statistical operator for the quantum object.

Proof. Since the characteristic projector $(1 \otimes Q_2^{(k)})$ of $(1 \otimes B_2)$ belongs to O_{12}^{H} (cf (14)), we can immediately evaluate with the help of (19) the probability of this quantum event in an arbitrary hybrid state M, given by (17). We thus obtain w_k (as given in (17)) as our result. Since a definite-b hybrid state by definition predicts one of the mentioned characteristic events say $(1 \otimes Q_2^{(k_0)})$, with certainty, M is such a state if and only if $w_k = \delta_{k,k_0}$. This completes the proof in view of the fact that for all values of k for which $w_k = 0$ in its general form (17), zero takes the place of the corresponding statistical operator. □

One should note that the definite-pointer-position hybrid states $M^{(k_0)}$ given by (20) *incorporate the cut* (cf section 3) and contain the relevant information on the pointer position on the subject side, which here is the classical half of $M^{(k_0)}$.

Now we can answer the previous question about a possible relevant decomposition.

Theorem 1. Let M be an arbitrary hybrid state given by (17). It *decomposes* as follows into definite-pointer-position hybrid states $M^{(k)}$ (of the form (20) *mutatis mutandis*):

$$M = \sum_k w_k M^{(k)}. \tag{21}$$

Here w_k and also the $\rho_1^{(k)}$ appearing in the $M^{(k)}$ states (when $w_k > 0$) are taken from the explicit form (17) of M. Decomposition (21) is *unique.*

Proof. The validity of (21) is a straightforward consequence of the way in which one evaluates a convex or a σ-convex combination in the set of all hybrid states $\mathcal{M}_{\mathrm{Q}}$ (see (18a) and (18b)). As to the claim of uniqueness, let us assume that there exists another decomposition:

$$M = \sum_k v_k M'^{(k)}$$

where

$$\forall k\colon v_k \geqslant 0, \sum_k v_k = 1$$

and

$$\forall k, v_k > 0\colon \qquad M'^{(k)} \equiv \{(\delta_{k',k}\rho_1'^{(k)}, \delta_{k',k}) : \forall k'\}$$

with some statistical operators $\rho_1'^{(k)}$. Then the rules (18a) and (18b) imply

$$M = \{(\rho_1'^{(k)}, v_k) : \forall k\}$$

where, for the k values for which $v_k = 0$, by definition $\rho_1'^{(k)} \equiv 0$. However M is uniquely determined in terms of the entities on the right-hand side of (17), that is a difference in any of these entities gives a distinct hybrid state. Hence,

$$\forall k\colon\ v_k = w_k$$

and

$$v_k \neq 0 \Rightarrow \rho_1'^{(k)} = \rho_1^{(k)}. \qquad \square$$

The key decomposition (21) can be derived in terms of Jauch's classes of statistical operators ρ_{12}. Actually, it has been obtained in this way by Jauch (1964, 1968) and by the author (extension and modification of Jauch's theory in Herbut 1986a).

In section 8 we discuss to what extent decomposition (21) answers the need for objectification of measurement.

7. The pointer observable as a beable

In this section we explore another way towards objectification by assuming that QM gives an *incomplete* statistical description of individual quantum systems. Moreover, we stipulate that the pointer observable $(1 \otimes B_2)$ of the composite quantum-object-plus-MI system is a *beable.*

In order to put the latter assumption in sufficient detail and with sufficient precision, we assume that we have an arbitrary quantum state ρ_{12} represented empirically by a laboratory ensemble of N composite systems. Then, to begin with, our beable stipulation has the following two aspects.

(i) In its *individual-system aspect* it reads that each of the N individual composite systems has a definite pointer position b_k prior to the measurement of any composite-system observable C_{12}.

(ii) In the *ensemble aspect* it is assumed that $(C_1 \wedge f(B_2)) \in O_{12}$ (cf (12)) is measured in ρ_{12}, and that $\tilde{N}_k$ is the number of systems that had b_k *prior to the measurement* and gave the 'wrong' result $(c_n, f(b_{k'}))$, $k' \neq k$, in the measurement. Then it is stipulated that

$$\left(\sum_k \tilde{N}_k\right) \Big/ N \to 0 \qquad \text{when } N \to \infty$$

that is that the measurement 'reads off' the prior-to-measurement value b_k of the beable $(1 \otimes B_2)$, that is it finds the 'right' result, with statistical certainty. By this the most that depends on the choice of the object-observable C_1 and on its obtained value c_n is the choice of the individual systems on which the value b_k of the beable fails to be communicated correctly to the MI that measures $(C_1 \wedge f(B_2))$, $(\Sigma_k \tilde{N}_k$ is the negligible number of such systems).

One should note that the second requirement implies partial locality of B_2: with statistical certainty, the pointer positions b_k found on the individual systems do not depend on the choice of C_1 in the measurement of $(C_1 \wedge B_2)$. This is an important point regarding so-called contextuality in the hidden-variable theories that exist in the literature (Shimony 1984, Djurdjević *et al* 1990).

Besides these requirements, we have to reconcile the distribution of the b_k values over the N systems with the statistical prediction implied by ρ_{12}:

(iii) Let N_k be the number of systems that have the pointer position b_k prior to measurement. Then, on account of (i), $\Sigma_k N_k = N$, and we require

$$\forall k: \lim_{N\to\infty} N_k/N = w_k \equiv \mathrm{Tr}_{12}\,\rho_{12}(1\otimes Q_2^{(k)})$$

where $Q_2^{(k)}$ is the characteristic projector of the pointer observable B_2 corresponding to the pointer position b_k.

Now, let us relate our beable approach to the idea of collapse (on which objectification rests). Let N'_k be the number of systems in the laboratory ensemble at issue that produce the result (c_n, b_k) with arbitrary c_n but fixed b_k in the measurement of the given observable $(C_1 \wedge B_2)$ (no matter if b_k is 'right' or 'wrong'). Let $\bar{N}'_k$ be the number of those among them that had $b_{k'}$, $k' \neq k$, prior to the measurement, that is that gave a 'wrong' result. Finally, let $\bar{N}_k$ be the number of those among the N_k systems that had b_k prior to measurement (cf (iii)), that also give a 'wrong' result $(c_n, b_{k'})$, $k' \neq k$. Then

$$N'_k - \bar{N}'_k = N_k - \bar{N}_k \tag{22}$$

(both sides give the number of systems that had b_k before the measurement and gave the 'right' result). Since $\bar{N}'_k \leqslant \Sigma_{k'}\bar{N}_{k'} \geqslant \bar{N}_k$, we have

$$\lim_{N\to\infty} \bar{N}'_k/N = \lim_{N\to\infty} \bar{N}_k/N = 0 \tag{23}$$

(cf (ii)). Hence, it follows from (22) that

$$\lim_{N\to\infty} N'_k/N = \lim_{N\to\infty} N_k/N = w_k \tag{24}$$

(cf (iii)).

Now we are prepared to give precise formulation of and to prove the basic result of this section.

Theorem 2. Let ρ_{12} be an arbitrary statistical operator describing the quantum state of a composite (quantum-object-plus-MI) system. We assume that a concrete laboratory ensemble of N composite systems is given that gives an empirical representation of ρ_{12}. For each k, N_k of the systems have b_k of B_2 prior to measurement (cf (i) and (iii)). The state represented empirically by the corresponding subensemble will be denoted by $\bar{\rho}_{12}^{(k)}$. (We refer to it as a 'substate' of ρ_{12}. If it is not a statistical operator, we call it a non-quantum mechanical state.)

Let C_1 be an arbitrary quantum-object observable that has a purely discrete spectrum. Then, for the measurement of $(C_1 \wedge B_2)$ we have the following statistical prediction:

$$\langle C_1 \otimes 1\rangle_{\bar{\rho}_{12}^{(k)}} = \mathrm{Tr}_1\,\rho_1^{(k)} C_1 = \langle C_1 \otimes 1\rangle_{M^{(k)}} \tag{25}$$

where the average is denoted by $\langle\ldots\rangle_{\bar{\rho}_{12}^{(k)}}$ in the mentioned definite-b_k substate $\bar{\rho}_{12}^{(k)}$, the collapsed state $\rho_1^{(k)}$ is given by (16), and the hybrid state $M^{(k)}$ is defined by (20) (with $\rho_1^{(k)}$ instead of ρ_1).

Thus, theorem 2 establishes that the non-quantum mechanical definite-b_k substates of ρ_{12} are actually correctly described by $M^{(k)}$, the definite b_k substates of the hybrid state M that corresponds to ρ_{12} (cf (20), (21) and (17) with (16) and (15)). At least this is true as far as the coincidence observables $(C_1 \wedge B_2)$ that is the instrument version of macroscopic complementarity, is concerned. Further, let us note that, as is easily seen, if and only if $[\rho_{12}, (1\otimes Q_2^{(k)})]=0$, the substate $\bar{\rho}_{12}^{(k)}$ is a statistical operator. It is then of the form (cf (30) below):

$$\bar{\rho}_{12}^{(k)}=(1\otimes Q_2^{(k)})\rho_{12}/\mathrm{Tr}_{12}(1\otimes Q_2^{(k)})\rho_{12}.$$

In this case (25) is trivially seen to be valid.

Proof of theorem 2. To begin with, we restrict ourselves to bounded C_1. For a fixed k value, let $i=1,2,\ldots,N_k$, enumerate in some arbitrary fixed order the individual composite systems in the definite-b_k subensemble prior to the measurement of $(C_1\wedge B_2)\in O_{12}$. Then, after the measurement,

$$\langle C_1\otimes 1\rangle_{\bar{\rho}_{12}^{(k)}}=\lim_{N\to\infty}\left(\sum_{i=1}^{N_k} c_i\right)N_k^{-1} \tag{26}$$

where $(c_i, b_{k'})$ is the result of the measurement on the ith composite system (with $b_{k'}=b_k$ or $b_{k'}\neq b_k$ though all N_k systems had b_k before the measurement).

We have to go over to the observed relative frequency when $(C_1\wedge B_2)$ is measured. Hence, we must take into account the possible 'wrong' results of b_k.

Let j enumerate (in some arbitrary but fixed order) the values of C_1 when the measurement of $(C_1\wedge B_2)$ gives (c_j, b_k). We have (in the notation used above):

$$\sum_{j=1}^{N'_k} c_j=\sum_{l=1}^{(N_k-\bar{N}_k)} c_l+\sum_{m=1}^{\bar{N}'_k} c_m$$

where l enumerates the 'right' results b_k among the N'_k observed ones, m enumerates the 'wrong' ones among them, and (22) has been utilized. On the other hand,

$$\sum_{l=1}^{(N_k-\bar{N}_k)} c_l=\sum_{i=1}^{N_k} c_i-\sum_{n=1}^{\bar{N}_k} c_n$$

where n enumerates those of the definite-b_k systems (prior to measurement) that give a 'wrong' result: $b_{k'}$, $k'\neq k$. Altogether,

$$\sum_{j=1}^{N'_k} c_j=\sum_{i=1}^{N_k} c_i-\sum_{n=1}^{\bar{N}_k} c_n+\sum_{m=1}^{\bar{N}'_k} c_m. \tag{27}$$

Denoting by $|C_1|$ the norm of the bounded operator C_1, we have further:

$$\left|\sum_{n=1}^{\bar{N}_k} c_n/N\right|\leqslant N^{-1}\sum_{n=1}^{\bar{N}_k}|c_n|\leqslant(\bar{N}_k/N)|C_1|$$

$$\left|\sum_{m=1}^{\bar{N}'_k} c_m/N\right|\leqslant N^{-1}\sum_{m=1}^{\bar{N}'_k}|c_m|\leqslant(\bar{N}'_k/N)|C_1|.$$

In view of (23), that is on account of the fact that the 'wrong' results occur, by stipulation, on a negligible subset of systems, it follows that

$$\lim_{N\to\infty}\sum_{n=1}^{\bar{N}_k} c_n/N=\lim_{N\to\infty}\sum_{m=1}^{\bar{N}'_k} c_m/N=0. \tag{28}$$

Taking into account (24), (27) and (28), and the fact that $w_k^{-1}=\lim_{N\to\infty}N/N_k$, we transform (26) as follows.

$$\langle C_1\otimes 1\rangle_{\bar{\rho}_{12}^{(k)}}=w_k^{-1}\lim_{N\to\infty}\sum_{i=1}^{N_k} c_i/N=w_k^{-1}\lim_{N\to\infty}\sum_{j=1}^{N'_k} c_j/N=w_k^{-1}\,\mathrm{Tr}_{12}\,\rho_{12}(C_1\otimes Q_2^{(k)}).$$

The last step is due to the fact that one has $c_j\delta_{k',k}$ as the result for $(C_1\otimes Q_2^{(k)})$ when one obtains $(c_j, b_{k'})$ in the measurement of $(C_1 \wedge B_2)$ on the entire quantum ensemble that empirically represents ρ_{12}.

Finally,

$$\langle C_1\otimes 1\rangle_{\bar\rho_{12}^{(k)}} = \mathrm{Tr}_1\, C_1\rho_1^{(k)}$$

in view of (16). This further equals $\langle C_1\otimes 1\rangle_{M^{(k)}}$ due to (19) and $1=\Sigma_k Q_2^{(k)}$.

This proof covers all cases when C_1 is a projector (quantum event). For any more general observable C_1 (bounded or unbounded) with a purely discrete spectrum (a more general C_1 cannot be exactly measured) the average values are linear combinations of the average values of the characteristic projectors with the corresponding characteristic values as the coefficients (with series instead of sums if the spectrum is infinite). Hence, (25) generalizes to any C_1 of interest. □

In theorem 2 we have made use of the extended set $\bar S_{12}$ of states that comprises besides all the quantum mechanical states ρ_{12} also all non-quantum mechanical definite-b_k substates $\bar\rho_{12}^{(k)}$ of the quantum mechanical states ρ_{12}. In all the states of $\bar S_{12}$ the expectation value of any observable $(C_1\otimes Q_2^{(k)})$ from O_{12}^{H} (given by (14)) is defined. Hence, the equivalence relation due to restriction to O_{12}^{H} (that was in S_{12} defined by (13)) can be extended to $\bar S_{12}$.

Theorem 3. Let $\bar\rho_{12}^{(k)}$ be a definite-b_k non-quantum mechanical substate of a quantum mechanical state ρ_{12}. One has

$$\langle C_1\otimes Q_2^{(k')}\rangle_{\bar\rho_{12}^{(k)}} = \langle C_1\otimes Q_2^{(k')}\rangle_{\rho_{12}^{(k)}} \tag{29}$$

for every Hermitian operator C_1 with a purely discrete spectrum and for every value of k' and k. Here $\rho_{12}^{(k)}$ is by definition the so-called Lüders (1951) projection of ρ_{12} (cf Messiah 1961):

$$\rho_{12}^{(k)} \equiv (1\otimes Q_2^{(k)})\rho_{12}(1\otimes Q_2^{(k)})/\mathrm{Tr}_{12}(1\otimes Q_2^{(k)})\rho_{12}. \tag{30}$$

(Note that the existence of $\bar\rho_{12}^{(k)}$ requires that $w_k \equiv \mathrm{Tr}_{12}(1\otimes Q_2^{(k)})\rho_{12} > 0$.)

Hence, $\bar S_{12}/\sim$ and $S_{12}/\sim$ are naturally isomorphic. Actually, they differ only in the fact that some classes (elements) of the former quotient set contain besides quantum mechanical also non-quantum mechanical states.

Proof. As evident from the definition (26) and (25), we have

$$\langle C_1\otimes Q_2^{(k')}\rangle_{\bar\rho_{12}^{(k)}} = \delta_{k',k}\langle C_1\otimes 1\rangle_{\bar\rho_{12}^{(k)}} = \delta_{k',k}\,\mathrm{Tr}_1\,\rho_1^{(k)}C_1.$$

On account of (30), we further have

$$= \delta_{k',k} w_k^{-1}\,\mathrm{Tr}_{12}(C_1\otimes Q_2^{(k)})\rho_{12} = \delta_{k',k}\,\mathrm{Tr}_{12}(C_1\otimes 1)\rho_{12}^{(k)}.$$

The rest of the claims follow in a straightforward way. □

8. Concluding remarks

We now give a critical discussion of the sense in which and the degree to which the objectification problem of quantum measurement theory has been solved in this paper.

As stated in the introduction, attributing physical meaning to some basic quantum mechanical concepts cannot, as a rule, be separated from taking a stand in the positivist-realist controversy (overlap with philosophy). Therefore, our discussion will also be given from this standpoint.

In sections 2-6 the criterion of relevance of a composite-system observable was given by restriction to O_{12} (defined by (12)) or rather by O_{12}^{H} (given by (14)). The first requirement of objectification (see the introduction) is now satisfied by decomposition (21). It displays the hybrid state M as a mixture of definite-pointer-position hybrid states $M^{(k)}$. One should note that M is the statistical state of the composite system in the instrument version. It corresponds to $|\phi\rangle_{12}\langle\phi|_{12}$ that, in turn, describes the same final premeasurement state in the object version.

The mechanism of the collapse $M \rightarrow M^{(k)}$ is furnished by the subsequent measurement of the pointer observable. Thus, the second requirement of objectification is also satisfied. Unfortunately, this argument is, at best, acceptable only to the broader-minded positivist (cf the introduction).

The realist's difficulties with this objectification argument are numerous. However one of them seems to be the most significant: It has been shown in the literature (Furry 1936, Moldauer 1972) that among the left-out observables (that is among those that do not belong to O_{12}) the doubly incompatible ones are the most important. They are of the form $(C_1 \wedge C_2)$ (cf section 4), where both

$$[C_1, A_1] \neq 0 \qquad \text{and} \qquad [C_2, B_2] \neq 0$$

are valid (A_1 being the observable measured in the first place, and B_2 being the pointer observable). Subsequent measurement of doubly incompatible observables (on the final premeasurement state) may distinguish between the coherent state $|\phi\rangle_{12}\langle\phi|_{12}$ given by

$$|\phi\rangle_{12} = \sum_k w_k^{1/2} |\phi^{(k)}\rangle_{12} \tag{31}$$

(cf (9)) and the corresponding (incoherent) mixture

$$\rho_{12} \equiv \sum_k w_k |\phi^{(k)}\rangle_{12}\langle\phi^{(k)}|_{12}. \tag{32}$$

The states $|\phi\rangle_{12}\langle\phi|_{12}$ and ρ_{12} given by (32) belong to the same Jauch class, that is the corresponding hybrid state M is one and the same. But they differ by the interference of the terms, that shows up experimentally precisely in the measurement of some doubly-incompatible observables $(C_1 \wedge C_2)$.

To the realist all that is potentially detectable (and possibly more than that) refers to aspects of reality. He cannot accept the claimed physical relevance of decomposition (21) unless satisfactory explanation is given about the doubly incompatible coincidence observables. The arbitrary, anthropocentric restriction to O_{12} will not do for him.

As a partial answer to this difficulty, the mentioned decomposition (21) is replaced by

$$|\phi\rangle_{12}\langle\phi|_{12} = \sum_k w_k \bar{\rho}_{12}^{(k)} \tag{33}$$

in the extended set $\bar{S}_{12}$ of statistical states (the first requirement of objectification). Now the pointer positions are assumed to have definite values for the individual (1+2)-systems in spite of the fact that $|\phi\rangle_{12}\langle\phi|_{12}$ is quantum mechanically homogeneous (it is such in S_{12} but not in $\bar{S}_{12}$), and in spite of the fact that it contains the mentioned interference as part of reality. Decomposition (33) has been shown (in theorem 3) to be meaningful with respect to the observables from O_{12}^{H} (given by (14)).

The way in which the pointer observable is imagined as a beable suggests that (33) should hold also regarding the doubly incompatible observables. But this is only a conjecture. It has not been proved in this paper.

The point to notice is that the terms $\bar{\rho}_{12}^{(k)}$ in (33) will, no doubt, give *different* statistical prediction for the doubly incompatible observables (if they give any) than the quantum mechanical states $|\phi^{(k)}\rangle_{12}\langle\phi^{(k)}|_{12}$, that are fictitious or 'ghost' states from the point of view of $|\phi\rangle_{12}$. Namely, the $|\phi^{(k)}\rangle_{12}$ states appear in the $M^{(k)}$ hybrid states (cf theorem 1), but they are substates only in ρ_{12} given by (32), and not in $|\phi\rangle_{12}$. The statistical state that corresponds in the object version to M is not ρ_{12}; it is $|\phi\rangle_{12}\langle\phi|_{12}$.

One can understand how inteference and sharp pointer positions in the states $|\phi\rangle_{12} \leftrightarrow M$ (having the two versions in mind) can both be real for the same individual systems: what interfere are the 'ghost' states $|\phi^{(k)}\rangle_{12}\langle\phi^{(k)}|_{12}$, and the real states $\bar{\rho}_{12}^{(k)}$ *do not interfere.*

As to the second requirement of objectification, the transition $|\phi\rangle_{12}\langle\phi|_{12} \rightarrow \bar{\rho}_{12}^{(k)}$ is due to the very beable property of B_2. But this is only the formal part of the answer. Essentially, the beable approach envisages a subquantum stochastic mechanism for the change from the initial pointer position b_0 to b_k the final one (see Bell 1987).

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